An Effective Method to Construct Coefficients in CONTRAST and ESTIMATE Statements used in ANOVA.

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ABSTRACT

The CONTRAST and the ESTIMATE statements available in the GLM and the MIXED procedures in SAS/STAT® system are useful in testing the significance of preplanned linear combinations of treatment means and in estimating mean and SE of linear combinations of model parameters respectively. Constructing the CONTRAST/ESTIMATE coefficients can be tricky and is difficult to understand especially in factorial experiments with the information available in the literature. Thus, an easy to follow method, involving three steps is propossed herein to construct coefficients in CONTRAST/ESTIMATE statements. The three steps include: constructing parameter vectors for all treatment groups, estimating coefficients for the linear combinations, and transferring the coefficients in the CONTRAST/ESTIMATE statements. This method is demonstrated using four examples from single factor, factorial, and mixed model experiments.

INTRODUCTION

In ANOVA, multiple comparison tests are performed when there are no particular comparisons of specific interests needed. But in most situations, the treatment structure suggests specific comparisons even before conducting the experiment. These would be called 'preplanned' comparisons because the contrasts are determined by the structure of the treatments not by the data. These preplanned contrasts are constructed based on the objectives of the experiment. Thus, the choice of the contrast comparisons must be decided before running the experiment and examining the data. In many cases, in addition to testing the hypotheses, obtaining linear combinations of parameter estimates with the standard error (SE) are also required. The SE is useful in estimating confidence intervals for the parameter estimates.

PROC GLM AND PROC MIXED

Preplanned comparison and parameter estimates in SAS can be performed using the GLM (SAS Institute Inc. 1990) and the MIXED (SAS Institute Inc. 1992) procedures using the optional 'CONTRAST' and 'ESTIMATE' statements.

The 'CONTRAST' statement is used to perform an F test on a hypothesis that linear combinations of parameters equal to zero. The basic form of the contrast statement in PROC GLM (SAS Institute Inc. 1990):

CONTRAST 'label' effect name effect - coefficients / OPTIONS;

Where:
label : name of the contrast
effect : effect in the model statement
effect coefficients: contrast coefficients

The MIXED procedure fits a mixed linear model with both fixed and random effects. For a split-plot analysis, the PROC MIXED provides both correct analysis and parameter estimates and eliminate the need for the error term specification ((SAS Institute Inc. 1992) The basic form of the contrast statement in PROC MIXED (SAS Institute Inc. 1992):

CONTRAST 'label' FIXED effect name FIXED effect - coefficients |RANDOM effect name RANDOM effect coefficients / OPTIONS;

There is no limit to the number of contrast statements, but they should follow the model statement. The ordering of the contrast coefficients follows the alphabetical ordering of the levels of the classification variable. You don't have to include trailing zeros. For difficult to write decimal fractions, it is easiar to multiply all coefficients by the least common denominator to get rid of the fractions. This is valid because in hypothesis testing, we test that a linear combination of model effects (treatments) equals to zero and therefore multiplication by a constant does not change the outcome of the hypothesis test(Littell et.al 1991). We can avoid the use of fractions by the use of a DIVISOR statement.

The ' ESTIMATE' statements in PROC GLM and PROC MIXED can be used to estimate the parameters. The 'ESTIMATE' statement is exactly like the CONTRAST term. where we replace the 'CONTRAST' term with the 'ESTIMATE' statement. The output of an 'ESTIMATE' statement includes the value of the estimate, a standard error, a *t*-statistic for testing whether the estimate is significanltly different from zero and a *p-value* for the t-statistic. The *p-value* for the t-statistc from the 'ESTIMATE statement is same as the *p-value* of the *F-test* in the 'CONTRAST' statement.

Contrast coeficients for estimating the interaction effects are the product of the corresponding coefficients for the two main effects. If most of the levels in two factors do not interact, these no interacting levels of each set of factors can be best compared after averaging over the second set of treatment. This provides more power for comparing the effects of two or more factors (Littell et.al 1991). It is also helpful to determine whether the interaction occurs in the data involving all levels of the factors. If the interaction in a factorial experiment is caused by only one level of the Factor A, then all other levels of factor A could be still compared after averaging all the levels of factor B. This provides more power for comparing the remaining levels of the factor. Thus, preplanned comparisons using 'CONTRAST' and 'ESTIMATE' statements are are prefered over pair comparisons.

However, the constructions of the coefficients are tricky and tedious. For more complicated experiments such as a two-way factorials, constructing the contrast coefficients is not straightforward. Furthermore, easy to follow methods to construct these coefficients are lacking from the currently available statistical text books and publications. Thus, a three-step method to construct coefficients in CONTRAST/ESTIMATE statements is presented here. The three steps include: constructing parameter vectors for all treatment groups using the parameter model for the data, estimating coefficients for the linear combinations of the treatment means suggested by the contrast comparison or by the parameter estimation, and transferring the coefficients in the CONTRAST/ESTIMATE statements. This method is demonstrated using four examples from single factor, factorial, and mixed model experiments.

EXAMPLES

Example 1: The first example is a single factor experiment with sub samples in a completely randomized design. The detailed descriptions of the data and the interested group comparisons are presented in Table 1a. Means and the standard deviations of the six treatments are displayed in Fig. 1. SAS codes including the contrast coefficients to analyze this data are presented in Table 1b. Because of the presence of sub samples in the data, appropriate experimental error term is specified using the 'E=' option in testing for treatment effect and the contrast comparisons. The 'E=' option is not available in the 'ESTIMATE' statement, therefore, parameter estimates using the 'ESTIMATE' statement is not possible for this example.

Results of the analysis are presented in Table 1c. Treatment effect is highly significant and the temperature effect accounted for most of the treatment effecyt. This is because since the high T versus. low T contrast accounted for more than 75% of the variation in the treatment effect.

Following steps are involved in constructing these contrast coefficients. First construct a table where rows represent model parameters and columns represent six treatment means (Table 1 e). The model parameters include the intercept and six parameters for each treatment. The ordering of the model parameters should follow the alphabetical ordering of the levels of the treatment variable. The construction of table with the mean coefficients and the subsequant manipulation of these coefficients can be effectively created using a spreadsheet program.

Secondly, write the coefficients for each treatment mean as follows: All treatment means contain the intercept, therefore add 1 for the intercept parameter for all treatment means. Next, add 1 if the treatments contain the specific parameter, otherwise specify 0. After writing coefficients for all treatment means, derive the specific contrast coefficients by adding and/or subtracting the treatment means columns involved in the specific comparison. For example, to derive the coefficients for the 'high T vs. low T' contrasts, subtract sum of three low temperature treatment columns (LT12, LT8, LT16) from the sum of three high temperature columns (HT12, HT8, HT16). Construction of the other three contrasts can be performed similarly and are shown in Table 1e.

Example 2: This is a two-factor experiment in a completely randomized design. The detailed descriptions of the data and the interested group comparisons are presented in Table 2a. Means and the standard deviations of the 12 treatment combinations are displayed in Fig. 2. The SAS codes including the coefficients for the preplanned contrasts to analyze this data are presented in Table 2b.

Results of the 4 x 3 factorial analysis is presented in Table 2c. The drug effect is highly significant and the effects of disease and the two factor interactions are not significant at 5% level. Results of the four preplanned contrast comparisons and two parameter estimates are presented in the Table 2c. The 'CONTRAST' statement provides only *P-value* for F-test where as the 'ESTIMATE' statement gives parameter estimate, the standard error for the estimate, and the *P-value* for the t-test. Both *P-values* for the same comparison obtained from 'CONTRAST' and 'ESTIMATE' statements are the same. However, the 'ESTIMATE' statement is more informative.

The steps involved in constructing these contrast coefficients are: First, construct a table of interaction mean coefficients. The rows represent the model parameters including the intercept, 4 levels of drug, 3 levels of diseases, and 12 levels of drug x disease interactions. The columns represent the 12 interaction treatment means (Table 2 e). Ordering of the model parameters should follow the order of the factor in the 'CLASS' statement and the alphabetical ordering of the levels of each factor.

Next, write the coefficients for each interaction treatment means as follows: All treatment means contain intercepts, therefore add 1 for the intercept parameter for all treatment mean. Then add 1, if the interaction mean contains the specific factor level or their interaction parameter. If not, specify 0. After completing all treatment mean coefficients, derive the specific contrast coefficients by adding and/or subtracting the treatment means columns involved in the specific comparison. The computations for constructing the four CONTRAST coefficients are shown in Table 2e.

Example 3: This is a three-factor experiment in a randomized complete block design. The detailed descriptions of the data and the interested group comparisons are presented in Table 3a. Factor A (salinity levels) is a quantitaive factor with 3 equally spaced levels. The linear effects of salinity under two levels of Nitrogen and two levels of aeration are displayed in Fig. 3. The SAS codes including the coefficients for estimating the slopes and the intercepts are presented in Table 3b.

Results of the 3 x 2 x 2 factorial analysis is presented in Table 3c. The linear effect of salinity (A), main effects of nitrogen (B), and aeration (C), interacttion between A x B and B x C are significant. Significant A x B interaction indicates that the slope is influenced by the nitrogen level. Statistically insignificant A x C interaction revealed that the slope is not changed by the level of aeration. Thus, two slopes (for two nitrogen levels) and 4 intercepts (2 nitrogen x 2 aeration) need to be estimated. The results of the 'ESTIMATE' statements are presented in the Table 3c .

The steps involved in constructing these ESTIMATE coefficients are: i) Slopes: For each nitrogen level, the change in the response when the salinity level is increased by one unit averaging over the two aeration levels. 2) Intercepts: The amount of response for a given nitrogen and aeration level when the salinity level = 0.

First construct a table of treatment means coefficients. The rows in this table represent the model parameters including the intercept, level of salinity, 2 levels of nitrogen, 2 levels of aeration, 2 levels of A x B interaction, and 2 levels of B x C. The columns represent treatment combination means at a specific salinity level (Table 3 e). Ordering of the model parameters should follow the order of the factor in the 'CLASS' statement and the alphabetical ordering of the levels of the each factor. Next, write the coefficients for each treatment combinations mean as follows: All treatment means contain the intercepts therefore, add 1 for the intercept parameter in all treatment mean. For the salinity-linear term, specify the quantity of the salinity level specific to the mean. If a model term (e.g., Factor C) is not included in a mean (e.g., A=15, B=1), divide 1 by the number of levels in the factor which is not included in the mean (e.g., C=2) and specify the resulting coefficient (0.5) as the coefficient for the factor C (Table 3e). For a given mean (e.g., A15B1), derive the interaction parameters (A x B) by multiplying the coefficients for the two factors (e.g., A and B). After completing all treatment mean coefficients, derive the specific contrast coefficients by adding and/or subtracting the treatment means columns involved in the specific comparison. The computations for constructing the eight ESTIMATE coefficients are shown in Tables 3e and 3f.

Example 4: This is a 4 x4 split-plot experiment in a randomized completely block design. The detailed descriptions of the data and the interested group comparisons are presented in Table 4a. Means and standard deviations of the sixteen treatment combinations are presented in Fig. 4 The SAS codes using the PROC MIXED including the coefficients for the preplanned contrasts and parameter estimates are presented in Table 2b. PROC MIXED provides correct *F-test* for comparisons involving the main factors within a split-plot factor and gives the appropriate broad inference space standard errors for this data (SAS Institute Inc. 1992).

The results of the 4 x 4 split-plot analysis using the 'PROC MIXED' is presented in Table 4c. The seed lot effect (A) and the interaction between A and B (Chemical) are highly significant. The results of five preplanned contrast comparisons and the mean estimate of (A2, A3) are presented in Table 4c. Standard errors obtained from the MIXED procedure are *broad* inference space estimates which are suitable when inferences applies to the entire population from which the random effects are sampled (SAS Institute Inc. 1992).

Steps involved in constructing these contrast coefficients are: First construct a table of interaction mean coefficients, where the rows represent the model parameters including the intercept, 4 levels of seedlot, 4 levels of chemical, and 16 levels of seedlot x chemical interactions. The columns represent, 16 interaction treatment means (Table 4e). The ordering of the model parameters should follow order of the factor in the 'CLASS' statement and the alphabetical ordering of the levels of each factor.

Write the coefficients for each interaction treatment mean as follows: All treatment means contain the intercepts, therefore add 1 for the intercept parameter for all treatment means. Then add 1 if the interaction mean contains the specific factor level or their interaction parameter. If not, specify 0. After writing all treatment mean coefficients, derive the specific contrast coefficients by adding, subtracting, multiplying, and/or dividing the treatment means columns involved in the specific comparison. The computations for constructing the six CONTRAST coefficients are shown in Table 4f.

CONCLUSION

A three-step method presented in this paper makes it easiar to construct complicated 'CONTRAST' and 'ESTIMATE' coefficients in factorial experiments. A spreadsheet can be used to simplyfy the construction of the table of coefficients. This method is demonstrated using four examples from single factor, factorial, and mixed model experiments.

REFERENCES

Littell R.C., Freund R. J. , and Spector P. C. (1991) *SAS system for linear Models. Third Edition.* Cary, NC: SAS Institute Inc.

SAS Institute Inc. (1990) *SAS/STAT User's Guide, Version 6, Fourth Edition.* Cary, NC: SAS Institute Inc.

SAS Institute Inc. (1992) *SAS Technical Report P-229 SAS/STAT Software: Changes and Enhancements.* Cary, NC: SAS Institute Inc.

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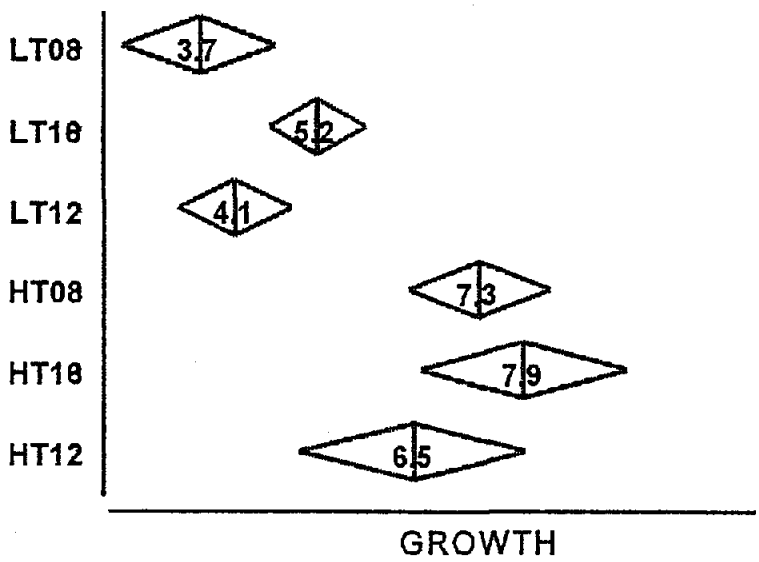
Table 1a. Description of the data used in the Example 1

Design:	Single Factor in a Completely Randomized Design with Subsamples
Source:	Principles and Procedures of Statistics By Steel and Torrie 1980 - page154
Response:	One-week stem growths of mint plants grown in nutrient solutions
Treatments:	6: HT/8h HT/12h HT/16h LT/8h LT/12h LT/16h
	Where:
	HT: High Temperature LT: Low Temperature
	8h: Eight h (short) Photoperiad
	12h: Tweleve h (neutral) Photoperiad
	16h: Sixteen h (long) Photoperiad.
Experimental Units: 3 Pots / Treatment	
SubSamples:	4 plants / Pot

Contrasts comparisons:

1) HT vs. LT 2) LT08 vs. LT16 3) HT08 vs. HT16 4) Ht16-HT08 = LT16-LT08

Fig 1. Display of the treatment means and standard deviation used in the Example 1.



LT08: Low Temperature-8h photoperiod LT16: Low Temperature -16 h photoperi od
LT12: Low Temperature-12h photoperiod HT08: High Temperature-8 h photoperiod
HT16: High Temperature-16 h photoperiod HT12: High Temperature-12 h photoperiod

Table 1b. SAS codes used to analyze the data in the Example 1.

```
PROC GLM DATA=MINT;
CLASS TREAT POT;
MODEL GROWTH=TREAT POT(TREAT);
TEST H=TREAT E=POT(TREAT);* experimental error = (pot (treat) to test the treatment effect;

* TREATMENT ORDER----------          HT12 HT16 HT8 LT12 LT16 LT8;

CONTRAST 'HIGH T VS LOW T'          TREAT  1    1    1   -1   -1   -1      /E=POT(TREAT);
CONTRAST 'LT08 VS. LT16'            TREAT  0    0    0    0   -1    1      /E=POT(TREAT);
CONTRAST 'HT08 VS. HT16'            TREAT  0   -1    1    0    0    0      /E=POT(TREAT);
CONTRAST 'HT08 VS. HT16'            TREAT  0   -1    1    0    0    0      /E=POT(TREAT);
CONTRAST 'HT16-HT8=LT16LT8'         TREAT  0    1   -1    0   -1    1      /E=POT(TREAT);
RUN;
```

Table 1c. A selected portion of the SAS output obtained from the analysis in the Example 1.

```
General Linear Models Procedure

Tests of Hypotheses using the Type III MS for POT(TREAT) as an error term

Source          DF    Type III SS       Mean Square    F Value      Pr > F
TREAT            5      179.64236         35.92847      16.69       0.0001

Dependent Variable: GROWTH

Tests of Hypotheses using the Type III MS for POT(TREAT) as an error term

Contrast                 DF    Contrast SS   Mean Square    F Value      Pr > F

high T vs low T           1      151.67014     151.67014      70.45      0.0001
LT08 VS. LT16             1       14.26042      14.26042       6.62      0.0244
HT08 VS. HT16             1        2.04167       2.04167       0.95      0.3494
HT16-HT8=LT16LT8          1        2.75521       2.75521       1.28      0.2800
```

Table 1e.Construction of the CONTRAST coefficients used in the Example 1.

Model Terms	Levels	Treatmet mean corlficients						Treatment contrast coelficients			
		HT12	HT16	HT8	LT12	LT16	LT8	1. HT-LT	2. LT8-LT16	3. HT8-HT16	4. HT16-HT8 = LT16-LT08
INTERCEPT	INTERCEPT	1	1	1	1	1	1	0	0	0	0
TRT	HT12	1	0	0	0	0	0	1	0	0	0
TRT	HT16	0	1	0	0	0	0	1	0	-1	1
TRT	HT8	0	0	1	0	0	0	1	0	1	-1
TRT	LT12	0	0	0	1	0	0	-1	0	0	0
TRT	LT16	0	0	0	0	1	0	-1	-1	0	-1
TRT	LT8	0	0	0	0	0	1	-1	1	0	1

1. HT-LT : High Temperature vs. Low Temperature = 1/3[(HT12 + HT16 +HT8) - (LT12 + LT16 + LT8)]
2. LT8-LT16: = LT8 - LT16
3. HT8-HT16:Difference between 8 and 16 h photoperiods under HIGH temperature = HT8 - HT16
4. HT16-HT8=LT16-LT8 : Differences between 8 and 16 h photoperiods under LOW ans HIGH temperatures are the same = HT16-HT8-LT16+LT8.

Table 2a. Description of the data used in the Example 2

Design: Factorial (4 x 3) in a Completely Randomized Design With missing obs.
Source: SAS STAT user's Guide The GLM Procedure Example 3 - page 622
Response: Observed from each subject.
Treatments: Factor A: Drug 4 levels : 1, 2, 3, 4.
Factor B: Disease 3 levels: 1, 2, 3
Experimental Units: 6 subjects / Treatment combinations

Pre-planned Contrasts/ Estimates
1). (Drug1+2)vs.(Drug3+4) 2). Dis3 vs. Dis2 for Drug2 3). Drug2 vs. Drug3 for Dis2
4). (Drug2/Dis2-Drug2/Dis3)= (Drug3/Dis2-Drug3/Dis3)

Fig 2. Display of the Drug x Disease interaction means and standard deviation used in the Example 2.

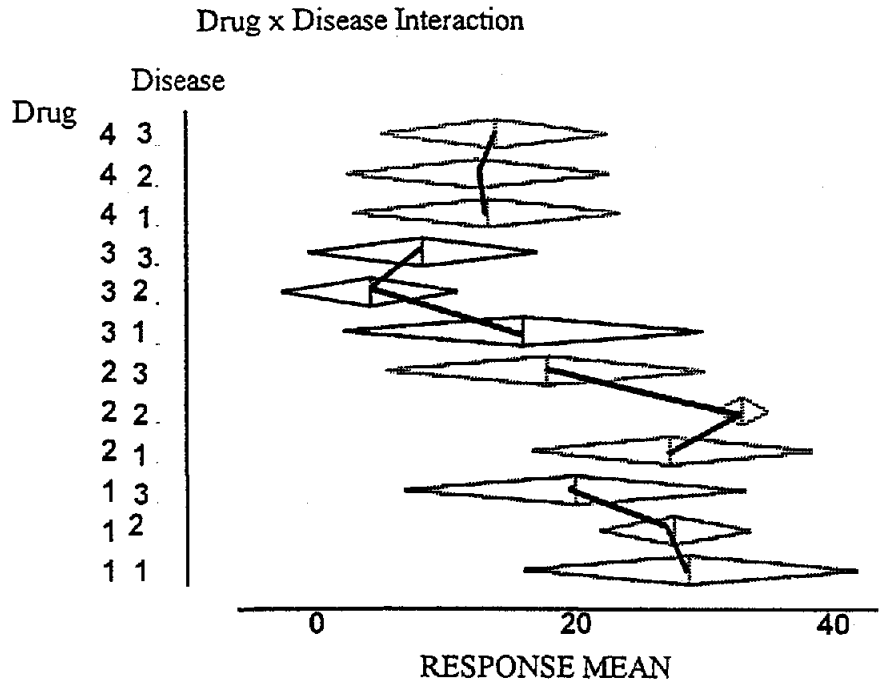


Table 2b. SAS codes used to analyze the data in the Example 2.

```
PROC GLM;
   CLASS DRUG DISEASE;
   MODEL Y=DRUG DISEASE DRUG*DISEASE;

CONTRAST 'DRUGS 1$2 VS. 3$4'     DRUG 3 3 -3 -3 DRUG*DISEASE 1 1 1 1 1 1 -1 -1 -1 -1 -1 -1;
CONTRAST 'DISEASE 3 VS.2 -DRUG2' DISEASE 0 1 -1 DRUG*DISEASE 0 0 0 0 1 -1 0 0 0 0 0 0;
CONTRAST 'DRUG 2 VS 3 - DIS 2'   DRUG 0 1 -1 0 DRUG*DISEASE 0 0 0 0 1 0 0 -1 0 0 0 0;
CONTRAST 'DRUG 2, 3 X DIS 2, 3'              DRUG*DISEASE 0 0 0 0 1 -1 0 -1 1 0 0;

ESTIMATE 'DRUG 2 VS 3 - DIS 2'   DRUG 0 1 -1 0 DRUG*DISEASE 0 0 0 0 1 0 0 -1 0 0 0 0;
ESTIMATE 'DRUG 2, 3 X DIS 2, 3'              DRUG*DISEASE 0 0 0 0 1 -1 0 -1 1 0 0;

RUN;
```

Table 2c. A selected portion of the SAS output obtained from the analysis in the Example2.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
DRUG	3	2997.4719	999.1573	9.05	0.0001
DISEASE	2	415.8730	207.9365	1.88	0.1637
DRUG*DISEASE	6	707.2663	117.8777	1.07	0.3958

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
DRUGS 1$2 VS. 3$4	1	2982.9343	2982.9343	27.01	0.0001
DISEASE 3 VS.2 -DRUG	1	564.2667	564.2667	5.11	0.0286
DRUG 2 VS 3 - DIS 2	1	1881.8000	1881.8000	17.04	0.0002
DRUG 2, 3 X DIS 2, 3	1	435.7551	435.7551	3.95	0.0530

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
DRUG 2 VS 3 dis 2	29.1000000	4.13	0.0002	7.05008094
DRUG 2, 3 X DIS 2,	19.4333333	1.99	0.0530	9.78394253

Table 2e. Construction of the CONTRAST coefficients used in the Example 2.

Model Terms	Level	Interaction mean coefficients												Linear Contrast Coefficients			
		A1B1	A1B2	A1B3	A2B1	A2B2	A2B3	A3B1	A3B2	A3B3	A4B1	A4B2	A4B3	1. (A1+A2)-(A3+A4)	2 A2B2-A2B3	3. A2B2-A3B2	4. A2B2-A2B3-A3B2+A3B3
INTERCEPT	INTERCEPT	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
Drug (A)	A1	1	1	1	0	0	0	0	0	0	0	0	0	3	0	0	0
Drug (A)	A2	0	0	0	1	1	1	0	0	0	0	0	0	3	0	1	0
Drug (A)	A3	0	0	0	0	0	0	1	1	1	0	0	0	-3	0	-1	0
Drug (A)	A4	0	0	0	0	0	0	0	0	0	1	1	1	-3	0	0	0
Disease (B)	B1	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
Disease (B)	B2	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
Disease (B)	B3	0	0	1	0	0	1	0	0	1	0	0	1	0	-1	0	0
A* B	A1B1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
A* B	A1B2	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
A* B	A1B3	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
A* B	A2B1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
A* B	A2B2	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
A* B	A2b3	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	0	-1
A* B	A3B1	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0
A* B	A3B2	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	-1	-1
A* B	A3B3	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	1
A* B	A4B1	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0
A* B	A4B2	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0
A* B	A4B3	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0

1) Drug1 &2 vs. Drug3&4 =(A1B1+A1B2+A1B3+A2B1+A2B2+A2B3)-(A3B1+A3B2+A3B3+A4B1+A4B2+A4B3)
2) Dis3 vs.Dis2 for Drug 2 =(A2B3-A2B2) 3) Drug2 vs. Drug3 for Dis2 = (A2B2-A3B2)
4) Interaction between Drugs 2 &3 and Diseases 2 &3 =(A2B2-A2B3-A3B2+A3B3)

Table 3a. Description of the data used in the Example 3

Design: Factorial (3 x 2x 2) in Randomized Complete Block Design
Source: Principles and Procedures of Statistics By Steel and Torrie 1980 - page202
Response: Dry weight of Biomass of a mash plant
Treatments: Factor A- Quantitative 3 levels :Salinity levels: 15, 30, 45 ppt
Factor B - 2 levels of Nitrogen : 0 , 168 kg N/ha.
Factor C - 2 levels of Aeration : None and Saturaion.
Blocks: 4 blocks / treatment combinations

Contrasts/ Estimates

1. Intercept, N=0 , Air=none
2. Intercept, N=168 , Air=none
3. Intercept, N=0 , Air=Saturation
4. Intercept, N=168, Air=Saturation
5. Slope, N=0, Air= mean of (None +Saturation)
6. Slope, N=168, Air= mean of (None +Saturation)
7. At 20 salinity and air=saturated condition, diff between N0 and N168
8. At 35 salinity and air=saturation condition, diff between N0 and N168

Analysis:- Using 'PROC GLM'

Fig 3. Display of the Salinity x N-level interactions for 2 levels of Aeration used in the example3.

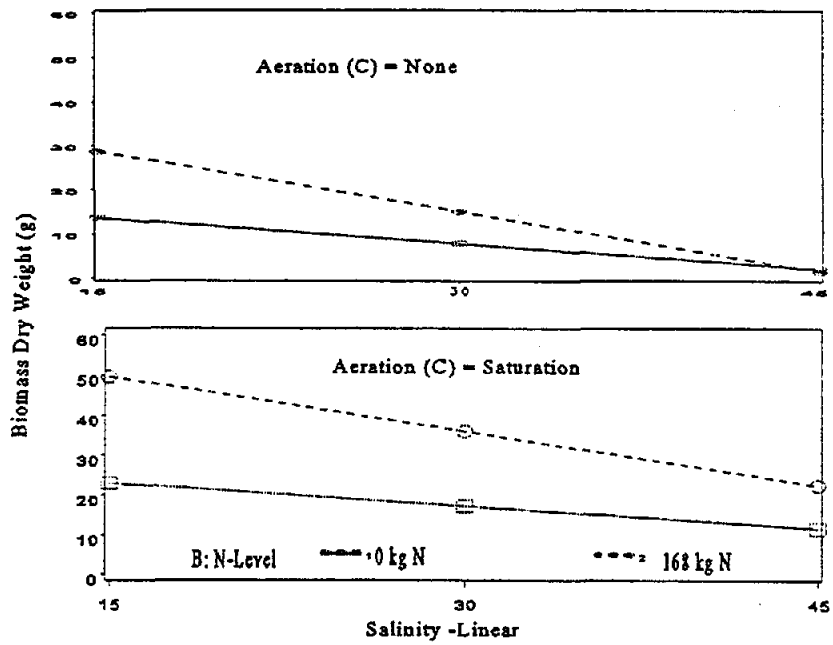


Table 3b. SAS codes used to analyze the data in the Example 3.

```
PROC GLM;    CLASS B C;
   MODEL RESP= AL B AL*B  C  B*C;      * AL - Quantitative variable-covariate;

ESTIMATE 'SLOPE1-B1' AL 1  AL*B 1;
ESTIMATE 'SLOPE1-B2' AL 1  AL*B 0 1;
ESTIMATE 'INTERCEPT-B1C1' INTERCEPT 1 B 1  C 1  B*C 1;
ESTIMATE 'INTERCEPT-B1C2' INTERCEPT 1 B 1  C 0 1 B*C 0 1;
ESTIMATE 'INTERCEPT-B2C1' INTERCEPT 1 B 0 1 C 1  B*C 0 0 1;
ESTIMATE 'INTERCEPT-B2C2' INTERCEPT 1 B 0 1 C 0 1 B*C 0 0 0 1;
ESTIMATE 'B1C1 VS B2C1 AT A=20' B 1 -1 AL*B 20 -20  B*C 1 0 -1 0;
ESTIMATE 'B1C1 VS B2C1 AT A=35' B 1 -1 AL*B 35 -35  B*C 1 0 -1 0;

RUN;
```

Table 3c. A selected portion of the SAS output obtained from the Example3 analysis.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
AL	1	2960.6513	2960.6513	27.45	0.0001
B	1	1401.4001	1401.4001	12.99	0.0008
AL*B	1	489.8450	489.8450	4.54	0.0390
C	1	2745.1875	2745.1875	25.45	0.0001
B*C	1	396.7500	396.7500	3.68	0.0619

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
slope1-b1	-0.3804167	-2.20	0.0335	0.17308652
slope1-b2	-0.9020833	-5.21	0.0001	0.17308652
intercept-b1c1	19.3208333	3.22	0.0025	5.99589283
intercept-b1c2	28.6958333	4.79	0.0001	5.99589283
intercept-b2c1	42.1625000	7.03	0.0001	5.99589283
intercept-b2c2	63.0375000	10.51	0.0001	5.99589283
b1c1 vs b2c1 at a=20	-12.4083333	-2.53	0.0151	4.89562599
b1c1 vs b2c1 at a=35	4.5833333	-1.04	0.3049	4.41285764

Table 3e. Construction of the ESTIMATE coefficients (1 to 6) used in the Example 3.

Model Terms	Level	B1 -slope estimation			B2 - Slope estimation			Intercepts estimation			
		B1A15	B1A16	1. B1 Slope	B2A15	B2A16	2. B2-Slope	3. B1C1Int	4. B2C1Int	5. B1C2Int	6. B2C2Int
INTERCEPT	INTERCEPT	1	1	0	1	1	0	1	1	1	1
Salinity-L (AL)	SL (QTY)	15	16	1	15	16	1	0	0	0	0
N-level(B)	B1(0)	1	1	0	0	0	0	1	0	1	0
N-level(B)	B2(168)	0	0	0	1	1	0	0	1	0	1
AL*B	AL*B1	15	16	1	0	0	0	0	0	0	0
AL*B	AL*B2	0	0	0	15	16	1	0	0	0	0
Aeration(C)	None (C1)	0.5	0.5	0	0.5	0.5	0	1	1	0	0
Aeration(C)	Saturation (C2)	0.5	0.5	0	0.5	0.5	0	0	0	1	1
B*C	B1*C1	0.5	0.5	0	0	0	0	1	0	0	0
B*C	B1*C2	0.5	0.5	0	0	0	0	0	0	1	0
B*C	B2*C1	0	0	0	0.5	0.5	0	0	1	0	0
B*C	B2*C2	0	0	0	0.5	0.5	0	0	0	0	1

1. Slope N=0 and Air=Mean of none and saturation = B1A16 -B1A15
2. Slope N=100 and Air=Mean of none and saturation = B2A16 -B2A15
3. Intercept N=0 Air=none B1C1Int
4. Intercept N=100 Air=none B2C1Int
5. Intercept N=0 Air=Saturation B1C2Int
6. Intercept N=100 Air=Saturation B2C2Int

Table 3f. Construction of the ESTIMATE coefficients (7-8) used in the Example 3.

Model Terms	Level	Interaction Means Coefficienst				Coefficients for Contrasts	
		B1C1A20	B2C1A20	B1C1A35	B2C1A35	B1C1-B2C1atA20	B1C1-B2C1at A35
INTERCEPT	INTERCEPT	1	1	1	1	0	0
Salinity-L (AL)	SL (QTY)	20	20	35	35	0	0
N-level(B)	B1(0)	1	0	1	0	1	1
N-leval(B)	B2(168)	0	1	0	1	-1	-1
AL*B	AL*B1	20	0	35	0	20	35
AL*B	AL*B2	0	20	0	35	-20	-35
Aeration(C)	None (C1)	1	1	1	1	0	0
Aeration(C)	Saturation (C2)	0	0	0	0	0	0
B*C	B1*C1	1	0	1	0	1	1
B*C	B1*C2	0	0	0	0	0	0
B*C	B2*C1	0	1	0	1	-1	-1
B*C	B2*C2	0	0	0	0	0	0

7. Difference between N0 and N168 at salinity=20 and air=saturation = B1C1A20 -B2C1A20
8. Difference between N0 and N168 at salinity=35 and air=saturation = B1C1A35 -B2C1A35

Table 4a. Description of the data used in the Example 4

Design:	Split-Plot (4 x4) in Randomized Complete Block Design
Source:	Principles and Procedures of Statistics By Steel and Torrie 1980 - page384
Response:	Yield of Oats in bushals per acre
Treatments:	Main-Plot Treatment (A)Seed Lot:
	a1=Viceland-1 a2= Vicland2 a3= Clinton a4= Branch
	Split-Plot treatment (B) Seed treatment with Chemicals:
	b1=Control b2=CeresanM b3=Panogen b4: Agrox
Blocks:	4 blocks / main-plot treatment

Contrasts/ Estimates

1. Seedlot1 vs.mean of lot2,3 2. Seedlot2and 3 vs. lot4 3. Trt1 vs trt2,3 in seedlot 1
4. Trt1 vs trt2,3 in seedlot 2 to 4.
5. seedlot vs. lot2,3 with no seed treatment. 6. Mean of seed lots 2 and 3.

Fig 3. Display of the Salinity x N-level interactions for 2 levels of Aeration used in the example3.

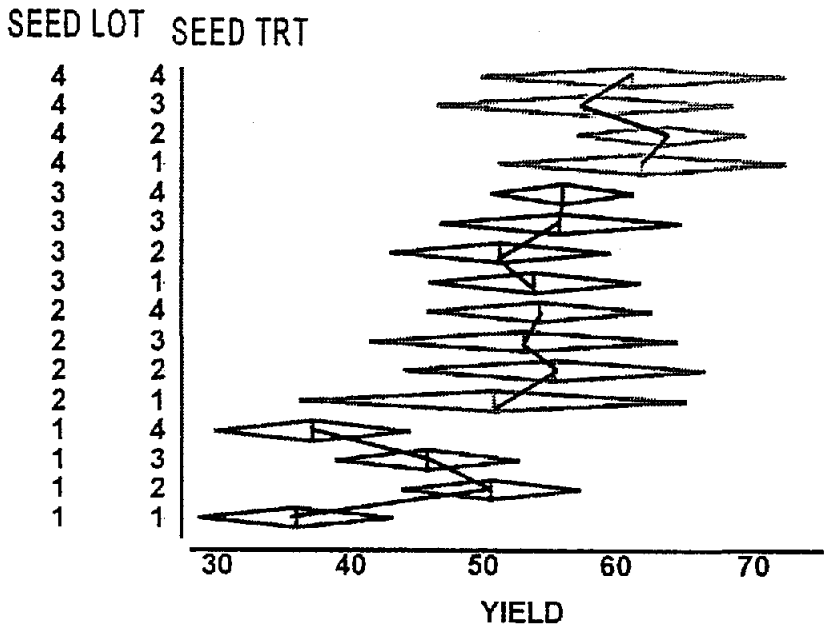


Table 4b. SAS codes used to analyze the data in the Example 4.

```
PROC MIXED DATA=OATS;
   CLASS A B REP;
   MODEL RESP=A|B;
   RANDOM REP REP*A;
* Broad Inference Space Estimates;
ESTIMATE 'A1 VS MEAN(A2,A3)'          A 8 -4 -4 0 A*B 2 2 2 2 -1 -1 -1 -1 -1 -1 -1 -1      /DIVISOR=8;
ESTIMATE 'A4 VS MEAN(A2, A3)'         A 0 -4 -4 8 A*B 0 0 0 0 -1 -1 -1 -1 -1 -1 -1 -1 2 2 2 2 /DIVISOR=8;
ESTIMATE 'B1 VS MEAN(B2,B3) AT A1'    B 2 -1 -1 0 A*B 2 -1 -1                                  /DIVISOR=2;
ESTIMATE 'B1 VS MEAN(B2,B3) AT A2-A4' B 6 -3 -3 0 A*B 0 0 0 0 2 -1 -1 0 2 -1 -1 0 2 -1 -1 0  /DIVISOR=6;
ESTIMATE 'A1 VS MEAN(A2,A3) AT B1'    A 2 -1 -1 0 A*B 2 0 0 0 -1 0 0 0 -1                      /DIVISOR=2;
ESTIMATE 'MEAN(A2,A3)'              INT 8 A 0 4 4 0 B 2 2 2 2 A*B 0 0 0 0 1 1 1 1 1 1 1 1    /DIVISOR=8;
RUN;
```

Table 4c. A selected portion of the SAS output obtained from the Example4 analysis.

The MIXED Procedure

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
A	3	9	13.82	0.0010
B	3	36	2.80	0.0539
A*B	9	36	3.21	0.0059

The MIXED Procedure

ESTIMATE Statement Results

Parameter	Estimate	Std Error	DDF	T	Pr > |T|
A1 vs MEAN(A2,A3)	-11.40000000	2.53782710	36	-4.49	0.0001
A4 vs MEAN(A2, A3)	7.21250000	2.53782710	36	2.84	0.0073
B1 VS MEAN(B2,B3) AT A1	12.18750000	2.75983563	36	-4.42	0.0001
b1 vs MEAN(b2,B3) At A2-A4	-0.57083333	1.59339184	36	-0.36	0.7222
A1 VS MEAN(A2,A3) AT B1	-16.33750000	3.48612764	36	-4.69	0.0000
MEAN(A2,A3)	53.85625000	3.98497912	36	13.51	0.0000

Table 4e. Constructing the parameter coefficients for the data used in the Example 4.

Model Terms	Level		Coefficients for the interaction means														
		a1b1	a1b2	a1b3	a1b4	a2b1	a2b2	a2b3	a2b4	a3b1	a3b2	a3b3	a3b4	a4b1	a4b2	a4b3	a4b4
Intercept		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Seedlot (A)	a1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	a2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
	a3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
	a4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
Chemical (B)	b1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
	b2	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
	b3	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
	b4	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
A x B	a1b1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	a1b2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	a1b3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	a1b4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	a2b1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	a2b2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	a2b3	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	a2b4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	a3b1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	a3b2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	a3b3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	a3b4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	a4b1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	a4b2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	a4b3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	a4b4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 4f. Constructing the broad inference space ESTIMATE coefficients for the data used in the Example 4.

Mosdel Terms	Level	Coefficients for the ESTIMATE statements					
		1. alvs a2&a3 mean	2. a4 vs. a2&a3 mean	3. b3 vs mean of(b2+b3) in al	4. bl vs mean(b2+b3) in a2-a4	5. al vs mean(a2+a3) in bl	6. mean (a2+a3)
Intercept		0	0	0	0	0	8
Seedlot (A)	al	8	0	0	0	2	0
	a2	-4	-4	0	0	-1	4
	a3	-4	-4	0	0	-1	4
	a4	0	8	0	0	0	0
Chemical (B)	bl	0	0	2	6	0	2
	b2	0	0	-1	-3	0	2
	b3	0	0	-1	-3	0	2
	b4	0	0	0	0	0	2
A x B	albl	2	0	2	0	2	0
	alb2	2	0	-1	0	0	0
	alb3	2	0	-1	0	0	0
	alb4	2	0	0	0	0	0
	a2bl	-1	-1	0	2	-1	1
	a2b2	-1	-1	0	-1	0	1
	a2b3	-1	-1	0	-1	0	1
	a2b4	-1	-1	0	0	0	1
	a3bl	-1	-1	0	2	-1	1
	a3b2	-1	-1	0	-1	0	1
	a3b3	-1	-1	0	-1	0	1
	a3b4	-1	-1	0	0	0	1
	a4bl	0	2	0	2	0	0
	a4b2	0	2	0	-17	0	0
	a4b3	0	2	0	-1	0	0
	a4b4	0	2	0	0	0	0

1. Al vs mean of (A2,A3) =1/8[(2*(A1B1+A1B2+A1B3+A1B4) -(A2B1+A2B2+A2B3+A2B4+A3B1+A3B2+A3B3+A3B4))]
2. A2 vs mean of (A2,A3) =1/8[(2*(A4B1+A4B2+A4B3+A4B 4) -(A2B1+A2B2+A2B3+A2B4+A3B1+A3B2+A3B3+A3B4))]
3. B1 VS mean of (B2,B3) at Al = 1/2[2*(A1B1)-(A1B2+A1B3)]
4. B1 VS mean of (B2,B3) at A2 TO A4 =1/6[2*(A2B1+A3B1+A4B1) - (A2B2+A3B2+A4B2+A2B3+A3B3+A4B4)]
5. Al YS mean of (A2,A3) at B1 =1/2[2*(A1B1)-(A2B1+A3B1)]
6. Mean of (A2,A3) = 1/8[(A2B1+A2B2+A2B3+A2B4)+(A3B1+A3B2+A3B3+A3B4)]