A Reassessment of the Monetary Approach to Exchange Rate Determination: A Multivariate Analysis of Difference-Stationary Time Series

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ABSTRACT

Previous attempts to validate the Monetary Approach to exchange rate determination were largely unsucessful. This paper claims that a more appropriate technique, that takes the issue of non stationarity into account, yields positive results. Data for the U.K. and the U.S. are subject to both univariate and multivariate analysis. The Monetary Approach, with some qualifications, is validated as a long-run relationship.

1 INTRODUCTION

The Bretton Woods System of fixed but adjustable exchange rates was established in 1945. After the System collapsed in 1973, the exchange rates between the U.S. dollar and the currencies of other industrialized nations were allowed to float freely. In response to this new institutional environment, the monetary approach to exchange rate determination was developed, refined and empirically validated in the early and mid 1970s. The 1980s decade, however, witnessed considerable volatility in the foreign exchange markets, which in turn triggered more interest on exchange rate theory in general, and on the monetary approach in particular. However, attempts to replicate the previous (favorable) empirical results with the new, more volatile, data were overtly unsuccessful. Indeed, the forecasting performance of the monetary models were shown to be no better, and sometimes worse, than the predictions obtained assuming that the spot exchange rate follows a random walk (see, e.g., Baillie and McMahon, 1989).

In section two of this paper we present a brief overview of the three leading versions of the monetary approach to exchange rate determination, including a critical evaluation of the empirical results reported by other researchers. Section three shows that each of the involved variables are difference-stationary or integrated, meaning that they need to be differenced before they can be described by stationary processes. The implications of nonstationarity are discussed in the context of exchange rate determination. Section four discusses nonstationarity from a multivariate point of view, and the concept of cointegration is shown to be relevant for the description of the variables. Cointegration, if present, implies that a linear combination of difference stationary time series is stationary, meaning that the series move together *in the long run*. It is shown that there is at least one cointegrating vector among the involved variables, which validates the model as a long run relationship, although the results suggest that the monetary model should be further refined. Finally, section five contains conclusions and suggestions for future research.

2 THE MONETARY MODEL

Exchange rates are merely the relative prices of assets, determined in organized markets where prices can be adjusted on an instantaneous basis to whatever 'the market' regards as the currently appropriate price. In this *monetary* or *asset* view the exchange rate is viewed as freely moving to equilibrate the international demand for and supply of stocks of money. This is in opposition to the so called 'Keynesian' way of modeling exchange rates, which considered them as freely moving to equilibrate the *flow* demand and supply of foreign currency[1]. The asset market approach assumes that capital is perfectly mobile internationally, and that the interest rate parity, to be defined below, holds.

2.1 The Flexible Prices Monetary Model

This model assumes that all goods prices are flexible, that capital is perfectly mobile, and that domestic and foreign assets are perfect substitutes. However, domestic money is demanded only by domestic residents and foreign money only by foreign residents. The strongest assumption is that money supplies and real incomes are determined exogenously. The model is given by the following :

$$\begin{aligned} s_t &= p_t - p_t^* \\ m_t - p_t &= k + \phi y_t - \lambda r_t \\ m_t^* - p_t^* &= k^* + \phi^* y_t^* - \lambda^* r_t \\ r_t - r_t^* &= f_t - s_t \\ f_t &= E_t s_{t+1} \end{aligned} \qquad (1.1 - 1.5)$$

where the variables are defined as follows:
s : natural logarithm of the spot exchange rate. The exchange rate is denominated in units of domestic currency per unit of foreign currency.

[1] In this Keynesian framework, interest focused on goods market flows. The 'equilibrium' exchange rate was assumed to be determined by the export and import of goods. A historical account of this and other approaches is given by Pentecost (1993).

f : natural logarithm of the forward exchange rate.
y: natural logarithm of real income.
r : nominal interest rate.

Asterisks denote foreign quantities, and the subscript t denotes time. Equation (1.1) is the Purchasing Power Parity (PPP) relationship in absolute form, implying that the goods market is in continuous (or short run) equilibrium, i.e., the dollar price of a basket of goods is the same in the U.S. and abroad. Equations (1.2) and (1.3) give the equilibrium conditions for the money market. Both ϕ, the income elasticity of money demand, and λ, the interest rate semi-elasticity of money demand are expected to be positive. Equation (1.4) is the is the Covered Interest Rate parity (CIP), which is a non-arbitrage condition that states that it is not possible to make infinite and riskless profits by speculating in the forward market. Equation (1.5) states that the forward rate is an unbiased predictor of the future spot rate. The operator $E_t(.)$ denotes an expectation conditional on information available up to time t.

Assuming that domestic and foreign money demand elasticities are the same, the first three equations give:

$$s_t = -(k-k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(r_t - r_t^*) \quad (1.6)$$

The exchange rate can be interpreted as the relative price of two currencies, determined in equilibrium by the money market. A higher interest rate differential will increase s_t (i.e. the domestic currency will depreciate) because a higher interest rate will decrease the amount of money demanded making the domestic currency relatively abundant[2]. Increases in the money supply will also depreciate the domestic currency, without affecting the real interest rate; i.e. there are no short run liquidity effects in this model. An increase in real income will appreciate the domestic currency because higher income translates into more transactions and higher demand for money, which makes domestic money relatively scarce.

The nominal interest rate differential can be endogeneized by making use of (1.4) and (1.5) in (1.6). Solving for S_t gives:

$$s_t = \left(\frac{1}{1+\lambda}\right) z_t + \left(\frac{\lambda}{1+\lambda}\right) E_t s_{t+1} \quad (1.7)$$

where

$$z_t = -(k-k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*)$$

Solving (1.7) forwards gives:

$$s_t = \left(\frac{1}{1+\lambda}\right) \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^j E_t z_{t+j} \quad (1.8)$$

According to (1.8), the current spot rate is a function of future discounted values of the fundamentals (i.e. money supply and real income). For example, expected changes in future real income will have an impact on the current spot rate, even if the current fundamental values have not changed. This is a common characteristic of asset prices, which reflect the fact that assets do not deplete, and therefore, their value today is affected by the (discounted) value the asset is expected to have in the future. Note also that (1.8) is able to explain the presence of high volatility in s_t even if z_t is observed to be stable. For a particular solution to the rational expectations problem in (1.8) consider replacing $E_t(s_{t+1})$ with an observed proxy. For that, note that taking conditional expectations of the first differences of the PPP relation (1.1) gives:

$$E_t \Delta s_{t+1} = E_t (\Delta p_{t+1} - \Delta p_{t+1}^*)$$

By making use of (1.4) and (1.5) we obtain the Flexible Prices Monetary Model :

$$s_t = -(k-k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda E_t(\Delta p_{t+1} - \Delta p_{t+1}^*) \quad (1.9)$$

For estimation purposes, expected inflation in (1.9) is approximated by the long-term government bond rate.

2.2 The Fixed Prices Monetary Model.

The assumption of short run equilibrium in the goods market was relaxed by Dornbusch (1976). In this model, goods prices adjust to a new equilibrium with a lag, due to costs of adjustment or lack of complete information. The PPP is still assumed to hold in the long run, so that the long-run effects of, say, an increase in the money supply, will be identical to those predicted by the flexible prices model. The model can be characterized by adding the following equations:

$$(d_t - d_t^*) = \gamma(y_t - y_t^*) - \sigma(r_t - r_t^*) + \omega(s_t - p_t - p_t^*) \quad (2.1)$$

$$(p_t - p_t^*) - (p_{t+1} - p_{t+1}^*) = \partial[(d_t - d_t^*) - (y_t - y_t^*)] \quad (2.2)$$

[2] This is in contrast to the Keynesian model with capital mobility, in which a higher interest differential causes a capital inflow which *appreciates* the domestic currency. See note (1).

where d_t is domestic demand for goods. (2.1) describes relative demand for goods as a function of relative real income, the interest differential and the terms of trade. Equation (2.2) states that the price adjustment is proportional to current excess demand.

Equation (1.1), the PPP, is replaced by the following two equations:

$$\bar{s}_t = \bar{p}_t - \bar{p}_t^* \tag{2.3}$$

$$E_t s_{t+1} - s_t = \alpha(\bar{s}_t - s_t) \tag{2.4}$$

where the upperscore denotes a long run equilibrium value for the corresponding variable. Equation (2.3) states that PPP holds in the long run, and (2.4) specifies the expected depreciation as proportional to the current period's exchange rate deviation from its long run equilibrium value. From (1.2)-(1.5), (2.1) and (2.2) we have:

$$(p_t - p_t^*) = -(k - k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(E_t s_{t+1} - s_t) \tag{2.5}$$

In a long run equilibrium, the expected depreciation is zero, so that $E_t(s_{t+1} - s_t)=0$ and from (2.6) we find the long-run equilibrium exchange rate:

$$\bar{s}_t = -(k - k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) \tag{2.6}$$

After some algebraic manipulations, we find a solution for the nominal exchange rate:

$$s_t = b_0 + b_1(m_t - m_t^*) + b_2(y_t - y_t^*) + b_3(p_{t-1} - p_{t-1}^*) \tag{2.7}$$

where b_0, b_1, b_2 and μ are functions of parameters. Equation (2.7) actually gives the rational expectations solution under perfect foresight, as was done in Dornbusch (1976).

2.3 The Real Interest Rate Differential Model.
This model, suggested in 1979 by Frankel (1993) constitutes an extension of Dornbusch (1976) which allows inflation to occur in the long run, and highlights the importance of the real interest rate in the determination of the real exchange rate. PPP is still assumed to hold in the long run, thus a long run equilibrium version of (2.7) would be:

$$\bar{s}_t = -(k - k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(\bar{\pi}_t - \bar{\pi}_{t+1}^*) \tag{3.1}$$

Where, as before, an upperscore indicates long run equilibrium, and π_t denotes expected inflation. Short run deviations from the equilibrium exchange rate are given by the inflation gap:

$$E_t s_{t+1} - s_t = \alpha(s_t - \bar{s}_t) + (\bar{\pi}_{t+1} - \bar{\pi}_{t+1}^*)$$

This, combined with (1.4) and (1.5), states that the exchange rate deviations from its long run equilibrium depend on the real interest rate differential:

$$s_t = -(k - k^*) + (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(\bar{\pi}_{t+1} - \bar{\pi}_{t+1}^*) + \frac{1}{\alpha}[(r_t - \bar{\pi}_{t+1}) - (r_t^* - \bar{\pi}_{t+1}^*)] \tag{3.2}$$

2.4 Empirical evidence.
Baillie and Selover (1987) have estimated the following equation, which includes the different monetary models as special cases:

$$s_t = \alpha_0 + \alpha_1(m_t - m_t^*) + \alpha_2(y_t - y_t^*) + \alpha_3(\pi_{t+1} - \pi_{t+1}^*) + \alpha_4[(r_t - \pi_{t+1}) - (r_t^* - \pi_{t+1}^*)] \tag{4.1}$$

Where π_{t+1} is the long term interest rate (government bond yield) used as a proxy for expected inflation. Note that if $\alpha_3 > 0$ and $\alpha_4 < 0$ we obtain the flex price monetary model. If $\alpha_3 = 0$ and $\alpha_4 < 0$ we obtain the fix price monetary model, and finally, if $\alpha_3 > 0$ and $\alpha_4 < 0$ we obtain the real interest rate differential model. As shown in Baillie and Selover (1987, p.45), the results are disappointing, both in terms of goodness of fit and in terms of signs of the estimated coefficients. This is consistent with results reported elsewhere (e.g. Frankel, 1993). However, there is agreement (see Pentecost, 1993) that all three models find some support when applied to the early 1970's and the 1920's.

The estimation procedure outlined above suffers from serious deficiencies. In particular, the variables were assumed to be stationary, even though the regressions yield serially correlated residuals. In this regard it is important to distinguish whether the series are trend-stationary or difference-stationary. If they are stationary around a deterministic trend, then differencing the series would solve the problem. On the other hand, if they are difference-stationary, and we run regressions in levels, standard inference results do not hold. If nonstationarity is removed by taking first differences, important long-run information

will be lost. The proposed 'solutions' have consisted of taking first differences, or using the Cochrane-Orcutt estimation method, as was done in the estimations reported above.

Another issue is that of exogeneity. The equations estimated are considered reduced forms, and sometimes called semi-reduced forms. However, the 'exogeneity' of the right hand side variables is imposed and never tested. In the case of exchange rate modeling, it can be argued that the money supply, for example, is itself affected by the spot rate if Central Banks intervene precisely in response to movements in the spot rate. Similarly, real income may be affected by the exchange rate, as has been argued in relation to the 'strong dollar' of the early 1980's, which was blamed for 'deindustrializing' the U.S. This simultaneity problem has been solved by making use of instrumental variables. However, the validity of the instruments is imposed and never tested. Also, the estimated models lack dynamic structure, in the sense that the use of lagged variables is almost absent. For example, the use of error correction mechanisms (lagged equilibrium errors), which allow for a rich dynamic structure, could improve the estimation results for these models.

In section three we formally investigate the possible non stationarity of the relevant variables. In section four a long run relationship is estimated using an algorithm that considers all variables as jointly dependent in order to avoid simultaneity and endogeneity problems.

3 UNIVARIATE ANALYSIS

Non stationarity in a time series may be due to either a deterministic time trend or to a unit root. For example, if we consider a variable such as the Gross Domestic Product, the above distinction will determine whether economic recessions have permanent consequences for the level of future GDP, or only temporary effects that are offset during the recovery. Unfortunately, the distinction can not be made from a finite sample. As pointed out by *inter alia* Hamilton (1994), for any unit root (i.e. difference -stationary) process there exists a stationary process that will be impossible to distinguish from the unit root representation for any given sample size T. The converse is also true. Interestingly, however, we can arrive at a testable hypothesis if we are willing to restrict further the class of processes to be considered. For example, if we use a first order autoregressive process, i.e. AR(1):

$$y_t = \rho y_{t-1} + \varepsilon_t$$

then the restriction H_0: $\rho=1$ is testable. There may be good reasons to restrict ourselves to consider only low-order autoregressive representations. Often, parsimonious models perform best, and autoregressions are much easier to estimate than moving averages (MA) processes, particularly MA with a root near-unity.

Two sets of unit root tests are implemented, the Augmented Dickey-Fuller tests (Dickey and Fuller, 1979) and the Phillips-Perron tests (Phillips and Perron, 1987). This tests are applied to U.S. and U.K. data for the period June 1973 - May 1993. "spot" is the natural logarithm of the exchange rate (dollar price of the sterling pound). "p" stands for consumer price index, "m" for money (M1), "y" is an index of real output, and "r" are interest rates (three-month Treasury Bill rate). The prefix "uk" or "us" indicates the country, and all variables are in natural logarithms, with the exception of the interest rate.

The Augmented Dickey-Fuller test is parametric, and consists of estimating the following regression:

$$y_t = \sum_{i=1}^{p-1} \zeta_i \Delta y_{t-i} + \alpha + \rho y_{t-1} + \delta t + \varepsilon_t$$

and testing whether $\rho=1$. The results are shown in Table 1 below. The first column shows the F-test for H_0:$\alpha=0$ and $\rho=1$. The second column shows the t-statistic for $\rho=1$. ZDF is given by $T(\rho-1)/(1-\sum_{i=1}^{p-1}\hat{\zeta}_i)$, where T is the sample size. The lag length "p" was chosen as the minimum value that yields white noise residuals. DW is the Durbin Watson statistic for the estimated regression. The last row shows the critical values for the various tests.

Table 1
Augmented Dickey-Fuller Tests

	F	t	ZDF	DW
spot	2.91	-2.24	-15.08	1.98
ukr	5.33	-3.23	-21.45	2.00
usr	2.51	-2.08	-19.98	1.99
uky	2.51	-2.17	-12.75	1.93
usy	4.79	-3.08	-19.00	2.04
ukm	2.07	-1.81	-10.69	1.97
usm	2.49	-2.22	-34.30	1.99
ukp	5.83	-2.82	-5.56	2.06
usp	1.91	-1.09	-3.39	2.04
5%	6.34	-3.43	-21.04	

The Phillips-Perron tests, called ZT and ZP, are nonparametric, and are computed according to the modified algorithm suggested by Hamilton (1994). The SAS (*) software code written for this paper is available from the author upon request. Table 2 presents the results for these tests. Again, the last row shows the critical values.

Table 2
Phillips-Perron Tests

	ZT	ZP
spot	-1.91	-7.34
ukr	-3.85	-28.12
usr	-1.92	-7.93
uky	-2.79	-14.58
usy	-2.69	-13.84
ukm	-2.81	-13.82
usm	-3.18	-19.18
ukp	-2.10	-2.61
usp	-0.98	-1.51
5%	-3.43	-21.04

In both tables, the null hypothesis of nonstationarity is rejected when the corresponding statistic is below the critical value. According to the Augmented Dickey-Fuller ZDF test, we are able to reject non stationarity only for the U.K. interest rate and the U.S. Money Supply. According to the t-test in Table 1, we are unable to reject nonstationarity in every case.

Table 2 shows that according to both Phillips-Perron tests, we are unable to reject the null of non stationarity only for the U.K. interest rate. We can conclude that, given the autoregressive representation adopted, only the U.K. interest rate can be considered a stationary variable. This finding may explain why previous research was unsuccessful in estimating a monetary model for the exchange rate: if the variables are difference-stationary, a regression will in general yield residuals which are also non stationary, which invalidates the results. The residuals will be stationary only if the variables move together in the long run. This last point will be examined in the following section.

4 MULTIVARIATE ANALYSIS

The previous section showed that the variables relevant to the monetary approach to exchange rate determination can be better described as being non stationary, implying that random shocks to these series will have persistent effects in the distant future. In this section we introduce the concept of Cointegration developed by Granger and Weiss (1983). If X_t is a vector of non stationary variables, then any linear combination of these variables, say $\alpha'X_t$, will in general be non stationary. However, if there exists a vector β such that $\beta'X_t$ is stationary, then β is called a cointegrating vector for X_t.

Each non stationary variable will drift over time, displaying non constant mean and variance. However, if cointegration is present, then these variables will move together *in the long run.*

Johansen and Juselius (1990) have proposed a Maximum Likelihood procedure that, first, tests for the number of cointegrating vectors (if there is any) and, second, estimates the cointegrating vectors. For further details, the reader is referred to the Johansen and Juselius paper. The variables considered are five: the spot rate, interest differential ($r-r^*$), relative money supplies ($m-m^*$), relative price levels ($p-p^*$) and relative income levels ($y-y^*$). The model to be estimated is:

$$\Delta x_t = \mu + \sum_{j=1}^{4} \Gamma_j \Delta x_{t-j} + \Pi x_{t-5} + u_t .$$

The lag lenght of the autoregression, in this case equal to five, was chosen to ensure randomness of the residuals. In this case X_t is a five-dimensional vector, μ is a 5x1 vector of unknown constants and the Γ_j and Π are 5x5 matrices. The matrix Π is factored such that $\Pi = \alpha\beta'$, where α and β are 5xr matrices. If r=0, then there are no cointegrating vectors, and a vector autoregressive (VAR) specification in first differences is appropriate. If r=5, then *every* linear combination of the 5 series is stationary, and therefore a VAR in levels is appropriate since all variables are stationary. If $0<r<5$, then r equals the number of cointegrating vectors. According to both the maximal eigenvalue and the likelihood ratio tests proposed by Johansen and Juselius, there is only one cointegrating vector β such that $\beta'X_t$ is stationary. It should be noted, however, that the estimated cointegrating vector is not unique, so that we normalize on the coefficient of the spot rate:

$$s_t = -2.197(r-r^*)_t - 20.826(y-y^*)_t + 10.592(m-m^*)_t - 6.075(p-p^*)_t$$

With the exception of the interest rate differential, all coefficients have the expected signs. The existence of a cointegrating vector validates the monetary approach in the sense that a long-run relationship has been found among the relevant variables. However, the fact that the interest differential has the wrong sign hints that the monetary approach should be refined. This unexpected finding may be interpreted as a low responsiveness of money demand to changes in interest rates, i.e., the coefficient "λ" in (1.2) and (1.3) would be negligible. This result gives some weight to the capital inflow mechanism: a relatively higher interest rate will attract foreign capital, which will appreciate the domestic currency (i.e. decreasing the spot rate). This would explain why ($r-r^*$) has a negative sign.

5 CONCLUSIONS

After reviewing the monetary approach, this paper has criticized previous statistical approaches on the grounds that they ignore, *inter alia*, the non stationary

nature of the relevant time series. It is formally shown that most of the series are non stationary, implying that they are characterized by non constant mean and variance, and that the effects of random shocks affecting these series will persist in the distant future. However, it is found that there exists a linear combination of these series which is stationary, meaning that these variables move together in the long-run. This validates the monetary approach as a long-run relationship. However, the results suggest that the monetary approach should be modified to accomodate the responsiveness of capital flows to interest rate differentials.

A natural extension of this paper would be to make use of the long-run relationship in a dynamic, short-run, specification. This inquiry is left for future research.

REFERENCES

Baillie, R. and McMahon, P. (1989), *The Foreign Exchange Market*, Cambridge: Cambridge University Press.

Baillie, R. and Selover, D. (1987), "Cointegration and Models of Exchange Rate Determination", *International Journal of Forecasting*, 3, 43-52.

Dickey, D. and Fuller, W. (1979), "Distribution of the Estimates for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, 74, 427-432.

Dornbusch, R. (1976), "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, 84, 1161-1176.

Frankel, J. (1993) *On Exchange Rates*, Cambridge: MIT Press.

Granger, C. and Weiss, A. (1983) "Time Series Analysis of Error-Correction Models", in: *Studies in Econometrics, Time Series and Multivariate Statistics*, ed. by Karlin, S., Amemiya, T. and Goodman, L., New York: Academic Press, 255-278.

Hamilton, J. (1994), *Time Series Analysis*, Princeton: Princeton University Press.

Johansen, S. and Juselius, K. (1990) "Maximum Likelihood Estimation and Inference on Cointegration With Applications to the Demand for Money", *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

Pentecost, E. (1993), *Exchange Rate Dynamics*, Aldershot: Edward Elgar.

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