SAS's Options - An Application of SAS in Financial Option Pricing

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Abstract[1]

This paper discusses how can we use SAS as a tool in solving the option pricing problem using numerical methods. The author uses a simple and intuitive approach to illustrate the essential elements of financial options and he has also explained how to do it. The user-defined function via macro processing capability is discussed and sample SAS code is enclosed. The techniques are generic and can be applied to a wide variety of situations and applications.

Introduction

Option is one of the most important financial instruments in the modern investment world. It has been extended to almost all other aspects of the financial world, including the stock option compensation. A correct pricing scheme is essential to every players involved in this market.

Background Information on Options

An "Option" is a contract that allows the holder to acquire a right, but not the obligation, to commit in a transaction.[2] The first exchange-based option trading was started in the Chicago Board of Trade in April 1973. In the same year, Fischer Black and Myron Scholes have published their path-breaking paper on option pricing and leading to the rapid development of financial derivative markets in the next decades.

Terminology

Underlying Asset

The underlying asset is the asset where the option contract will be based on.

Put versus Call Option

A call option allows the option holder to have a right to *buy* the underlying asset at a pre-specified price; a put allows the option holder to have a right to *sell* the underlying asset at a pre-specified price.

Strike Price

This is the price where the underlying asset will be transacted if it happens.

Risk-free Rate

In pricing, one of the most important concepts is risk-free borrowing and lending. In other words, it is the interest rate that an investor can borrow or lend with guaranteed return. Any other rates higher than this will involve risk premium.

Volatility

This is the degree of uncertainty of the value of the underlying asset. This is the most important factor in option pricing as this directly affect how likely it is for the option holder to exercise the option.

Time to Expiration

Every option has an expiration date; the end of the contractual agreement.

Dividend Yield

Since some stocks will distribute dividend during the lifetime of the options, the amount paid as dividend will affect the value of option.

American versus European Options

American option holders can exercise their options *anytime prior to expiration* while the European option holders can *only* exercise their options *at the expiration date.*

Real Options

The option concept can be extended to other application such as project valuations and business strategies. These non-financial option applications are called real options.

Option Pricing Model

Black-Scholes Model

Late Fischer Black and Myron Scholes in Black and Scholes (1973) developed the Black-Scholes option pricing model. This model allows us to price an option easily under certain conditions and assumptions. For details, please refer to the original article or any other finance textbook. Since this is the most widely used model, we will discuss only this model in this paper.

Formula

Call Option =

$$C(S,T;X) = S \cdot N(d_1) \cdot e^{-d \cdot t} - X \cdot N(d_2) \cdot e^{-r \cdot t}$$

Put Option

$$= P(S,T;X) = X \cdot N(-d_2) \cdot e^{-r \cdot t} - S \cdot N(-d_1) \cdot e^{-d \cdot t}$$

where

$$d_1 = \frac{\ln(S/X) + \left((r-d) + \sigma^2/2\right) \cdot T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$$

S	=	Index Level
X	=	Strike Price
r	=	Risk-free Rate
d	=	Dividend Yield
t	=	Time to Expiration
σ	=	Volatility

SAS Code

```
/************************************/

/*
Calculation of Call Option Price
*/

/************************************/

%macro callop(s, x, v, r, d, t);

      %local d1 d2 call;

      %let d1 = (log(&s/&x)
+( (&r-&d) + 0.5*(&v**2))*&t)
/ (&v * sqrt(&t));

      %let d2 = &d1 - &v * sqrt(&t);
.

      %let call = &s
* exp(-d*&t)*probnorm(&d1)
- &x * exp(-&r*&t)*probnorm(&d2);

      &call

%mend callop;
```

In this macro, the value of the variables will be captured from the calling statement. They are then put into the macro via the first line of the macro. The order of the variables should be exactly the same as in the call function, or else a wrong answer will be resulted.

The first line of the macro read the parameters into the main program. The `%local` declares three local variables for internal (within this macro) use. The `%let` performs variables manipulation and the last line `&call` return the result to the calling program.

The macro works like a SAS function, the calling program just needs to invoke it as if it were a built-in function. For example, it can be invoked as:

```
Call = %callop(100, 120, 0.3, 0.0001,
0.000054, 0.001);
```

or

```
Call = %callop(index, strike_p, vol,
r_f_rate, div_yld, time_t_m);
```

This technique allows us to create user-defined function and it can be used easily throughout the whole program. We can also store a library of

functions and use `%include` command to include them on demand. It is similar to C and Java in function/program library.

Implied Volatility

In the option pricing model, the theoretical option price can be calculated if stock price, strike price, risk-free rate, dividend yield, time to expiration and volatility are known. However, we only observe option price, not volatility. Unfortunately, there is no closed form solution for volatility i.e. we are not able to write down the volatility as a function of other parameter. Therefore, we need to use numerical methods to find out the theoretical volatility given that the other parameters are known. This theoretical value of volatility is called implied volatility.

Importance

Can price provide the most important information for all financial assets? The answer is "Yes" and "No". Price is probably the most important factor for all financial assets except option. It is because of the unique time decay characteristics of options. When the time to expiration is decreasing, an option is *less* likely to have a higher profit (since the loss is limited to the option premium - asymmetric payoff), so the option buyer should pay less for it. For other things being equal, an option tomorrow will cost less than today. As a result, we are unable to compare yesterday's price with today's price.

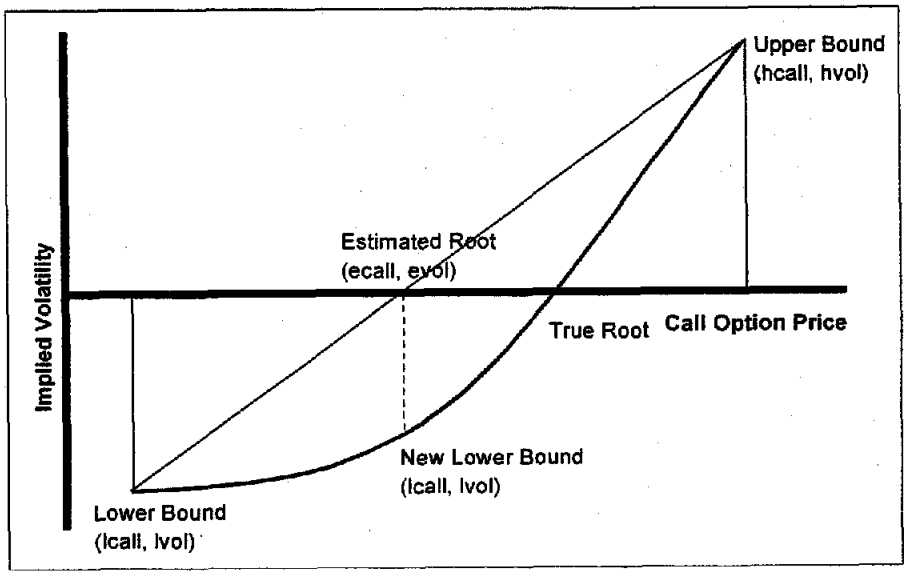
Using implied volatility, we are then able compare the "value" of an option consistently. Since we have already account for all of the relevant factors and we can compare the options across the boards![3]

Numerical Approximation

Unfortunately, from the Black-Scholes option pricing model, we are unable to calculate the volatility from the other parameters (there is no closed-form solution in mathematics jargon). Hence, we have to solve this problem numerically. Although the *Method of Bisection* is not the most efficient method in numerical computation, it is very simple to implement and easy to understand. Therefore, I use it for illustrative purpose.[4]

Method of Bisection

We know that the value of option will increase if the volatility of the underlying asset increase, for other things being equal. The method of bisection utilizes this property and tries to get the answer to a pre-specified degree to accuracy (convergence criteria).



Initially, an arbitrary upper and lower bound will be selected so that the option price of the upper bound volatility is positive and the option price of the lower bound volatility is negative. This procedure is very important because this will ensure that we will be able to find an answer. We will then interpolate the upper and lower bound to find an estimated option price and volatility. If the estimated result is of the same sign of the lower bound, we will use the estimated result as the new lower bound. If the estimated result is of the same sign of the upper bound, we will use the estimated result as the new upper bound. This process is repeated until the convergence criterion is met.

SAS Code

```
/*     For Call Option                    */

/*     Define upper bound - 40%           */

       hcv = 0.4;
       hcall
= %callop(s, x, hcv, r, d, t);

/*     Define lower bound - 10%           */

       lcv = 0.1;
       lcall
= %callop(s, x, lcv, r, d, t);

/*     Start the looping process,
convergence criteria = 0.0001 */

       do until( ((hcall-c)<0.0001) or
((c-lcall)<0.0001) );

ecv
= lcv
+ (c - lcall)*(hcv - lcv)
/(hcall - lcall);


ecall
= %callop(s, x, ecv, r, d, t);

             if (ecall > c) then
                   do;
                         hcv = ecv;
                         hcall = ecall;
                   end;
             else if (ecall < c) then
                   do;
                         lcv = ecv;
                         lcall = ecall;
                   end;

       end;

/*     End of implied volatility
calculation                          */
```

In the beginning of this program, we have defined the upper and lower bound as 40% and 10% respectively. This means that the implied volatility *has to be* laid between 10% and 40%. Otherwise, the convergence criteria can never be met and the program will loop forever. An improvement of this version should include upper and lower bound checking.

The convergence criterion is set to be 0.0001 (one basis point). If our numerical answer is less than one basis point from either the upper or lower bound, we are satisfied with this answer the it will exit the loop.

In fact, an intelligent upper and lower bound specification can help improving the computing time significantly. I have applied this method to stock index option pricing. Since both the intraday (within one trading day) and interday (between trading days) implied volatility normally will not fluctuate a lot. In fact, the best predictor of the current period implied volatility is the last period implied volatility (this is the first order autocorrelation). To improve the computation efficiency, we can set the upper and lower bound to the last period implied volatility plus and minus a certain percentage. For example:

Last period implied volatility = 25%

The new upper and lower bound is 27% (25% + 2%) and 23% (25% - 2%). The SAS program can then check to see if the option actually fall into this volatility bounds. In the above example, the new implied volatility falls inside the bounds most of the time.[5]

Other Important Information

Changes in Underlying Assets (delta)

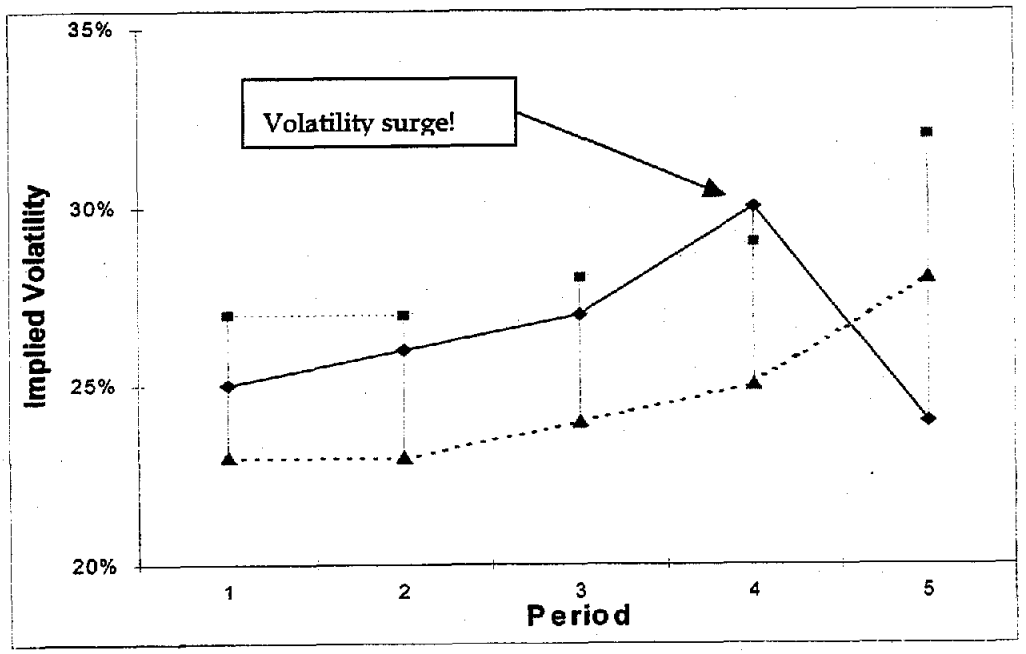
Delta is defined as the rate of change of the option price with respect to the rate of change of the underlying asset.

Delta can be used to determine the optimal hedge ratio, i.e. delta hedge. When an option portfolio has a zero delta, it means that the portfolio value will not be affected if there is an incremental change in the underlying asset. Please note that the value of delta is constantly changing and delta hedge is effective only if the changes are small.

Formula

$$Delta\ \Delta_{Call} = \frac{\partial c}{\partial s} = N(d_1)\cdot e^{-dt} > 0$$

$$Delta\ \Delta_{Put} = \frac{\partial p}{\partial s} = -N(-d_1)\cdot e^{-dt} < 0$$



SAS Code

```
/*
Calculation of the Delta of a call
Option
*/

%macro caldta(s, x, v, r, d, t);
      %local d1 delta;

      %let d1 = (log(&s/&x)
+(&r-&d + 0.5*(&v**2)) * &t)
/ (&v * sqrt(&t));

      %let delta = exp(-&d * &t)
* probnorm(&d1);

      &delta

%mend caldta;
```

```
/*
Calculation of the Delta of a Put
Option
*/

%macro putdta(s, x, v, r, d, t);
      %local d1 delta;

      %let d1 = (log(&s/&x)
+(&r-&d + 0.5*(&v**2)) * &t)
/ (&v * sqrt(&t));

      %let delta = -exp(-&d * &t)
* probnorm(-&d1);

      &delta

%mend putdta;
```

Applications

Option pricing is one of the most important fields in modern finance. It is also the cornerstone of the whole derivative market. Here I provide some sketches of the use of modern option pricing theory.

Financial Option Pricing

The derivative market has tremendous development in the last decade. In addition to traditional commodity and financial derivatives, we have derivatives on orange juice, electricity, bankruptcy and catastrophes. The options itself has also transformed from *plain vanilla* to *exotic*.

Mortgage Prepayment Model

Mortgage prepayment is a very important factor in mortgage pricing. It can be formulated using the option pricing concept. The lender of the loan/mortgage has written a call option to the borrower. The borrower will exercise this option (prepaid) if it is to his/her benefit. Some of the most sophisticated models are based on this simple idea.

Growth Opportunity for Uncertain Project

Another new application of option concept is "Real Option" concept in corporate finance. Since every opportunity should worth something, ignoring the value of an option to do something will lead to wrong conclusion. It can be used to explain why the "pure Internet" companies worth million of dollars — it is because the investors believe that those companies are in a good position to profit from the unknown prosperous future.

Conclusion

The application of SAS is unlimited. Here I have tried to provide a new application of SAS to financial instruments pricing. The technique used here, namely, the user-defined function approach, is generic and can be used in a wide variety of situation. The numerical method shown can also be used for other root solving problem.

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[2] You may think that financial option must be something invented in our rocket age. However, it can be traced back to the trading of olive-presses in ancient Greek. Aristotle has documented this in his *Politics*. The reference here is page 26 of Russell (1945).

[3] This is not an exactly correct statement. There are some discrepancies between theoretical and market values. They include "Smile Effect" (the implied volatility of both in-the-money and out-of-the-money options are higher than those of the at-the-money), "Show Effect" (the implied volatility of in-the-money/out-of-the-money call is higher than in-the-money/out-of-the-money put), and the "Team structure of volatility"(longer term options have a higher implied volatility). Mayhew (1995) has reviewed some empirical studies on smile effect.

[4] For a discussion on the pro and con of different estimation methods, see Brown (1990).

[5] Lai (1994) found that implied volatility of the Hang Seng Index Options in Hong Kong have a very strong first order autocorrelation. The best predictor for current period implied volatility seems to be the one-period lagged implied volatility, after corrected for AR(1) effect. Both Latane and Rendleman (1976) and Chiras and Manaster (1978) suggested that implied volatility could explain the future volatility of the underlying asset better than historical volatility.